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FINITE ELEMENT ANALYSIS OF MIXED CONVECTION HEAT TRANSFER ENHANCEMENT OF A HEATED SQUARE HOLLOW CYLINDER IN A LID-DRIVEN RECTANGULAR ENCLOSURE

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ABSTRACT

Mixed convection heat transfer inside a lid-driven enclosure along with a heated square hollow cylinder positioned at the center of the cavity has been analyzed numerically. Both the two vertical walls are maintained at uniform temperatures T_c, while the horizontal top and bottom walls are adiabatic. In addition, the left vertical wall of the cavity is allowed to move upward in its own plane at a constant velocity V_0, while the other walls remain stationary. The current study simulates a realistic system such as air-cooled electronic equipment with a heat component or an oven with heater. A Galerkin weighted residual finite-element method with a Newton-Raphson iterative algorithm is adopted to solve the governing equations. The computation is carried out for wide ranges of the cylinder location in the cavity. The numerical results are presented in the form of streamlines, isothermal lines, average Nusselt number at the heated surface, fluid temperature in the cavity, and drag force for different Richardson numbers. It is found that the flow field and temperature distribution strongly depend on the cylinder locations.

Keywords: Mixed Convection, Penalty Finite Element Method, Galerkin Method, Lid-Driven Cavity, And Square Hollow Cylinder.

1. INTRODUCTION

In recent years, mixed convection heat transfer enhancement in a lid-driven cavity is pertinent to much engineering and environmental applications such as heat exchanger, cooling of electronic equipments, nuclear reactors, chemical processing equipments and drying or geophysics studies, etc.

Barrier or a partition can be used to enhance heat transfer in cavities. To simulate the cooling of electronic equipments, Dagtekin and Oztop [1] inserted an isothermally heated rectangular block in a lid-driven cavity at different positions. They conclude that dimension of the body are the most effective parameter on mixed convection flow. Rahman *et al.* [2] made a numerical investigation on the effect of a heat conducting horizontal circular cylinder on MHD mixed convection in a lid-driven cavity along with joule heating. MHD mixed convection flow in a vertical lid-driven square enclosure including a heat conducting horizontal circular cylinder with Joule heating is performed in [3]. The numerical results indicated that the Hartmann number, Reynolds number and Richardson number had strong influence on flow and thermal fields in the enclosure. Billah *et al*. [4] performed a numerical study on the MHD mixed convection heat transfer enhancement in a double lid-driven obstructed enclosure, where the developed mathematical model was solved by using Galerkin weighted residual method of finite element formulation. It was found that the location of the block is one of the most important parameter on fluid flow, temperature fields and heat transfer characteristic. Reynolds and Prandtl numbers effects on MHD mixed convection in a lid-driven cavity along with joule heating and a centered heat conducting circular block has been analyzed in literature [5]. A numerical study on the effect of a heated hollow cylinder on mixed convection in a ventilated cavity was conducted by Mamun *et al* [6]. Authors showed the cylinder diameter has a significant effect on both the flow and thermal fields but the solid-fluid thermal conductivity ratio has a significant effect only on the thermal field. Very recently, Billah *et al* [7] have done a numerical investigation on fluid flow due to mixed convection in a lid-driven cavity having a heated circular hollow cylinder. It was found that the flow field and temperature distribution strongly depend on the cylinder diameter as well as a solid-fluid thermal conductivity ratio at the considered convective regimes.

To the best knowledge of the authors, no attention has been paid to the problem of mixed convection in a lid driven enclosure having a heated square hollow cylinder. The main aim of the current work is focused on conducting a comprehensive study on the effect of various flow and thermal configurations on mixed convection for different location of the heated square hollow cylinder in the cavity and Richardson number (*Ri*).

2. PHYSICAL MODEL

The present problem deals with a heated square hollow cylinder with thermal conductivity k_s located at different position of a square enclosure with sides of length L as shown in Fig.1. Both the two side walls are maintained at uniform constant temperatures T_c, while the horizontal top and bottom walls are adiabatic. The left vertical wall of the cavity is allowed to move upward in its own plane at a constant velocity V_0, while the other walls remain stationary. In addition, the radiation, pressure work and viscous dissipation are assumed to be negligible and Boussinesq approximation is assumed to be valid.

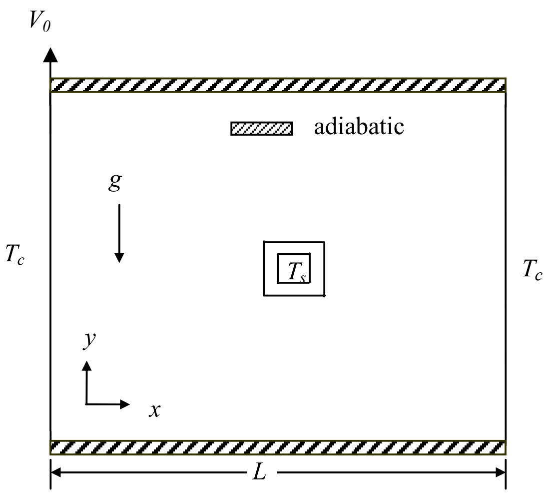

Fig 1. A schematic of the physical domain and boundary conditions

3. MATHEMATICAL FORMULATION

The flow is considered steady, laminar, incompressible and two-dimensional. The variation of fluid properties with temperature has been neglected with the only exception of the buoyancy term, for the Boussinesq approximation has been adopted. The governing equations and the boundary conditions are thrown in the dimensionless form using the following dimensionless variables:

$$X = \frac{x}{L},\ Y = \frac{y}{L},\ U = \frac{u}{V_0},\ V = \frac{v}{V_0},\ P = \frac{p}{\rho V_0^2}$$

$$D = \frac{d}{L},\ \theta = \frac{(T - T_c)}{(T_h - T_c)},\ \theta_s = \frac{(T_s - T_c)}{(T_h - T_c)}$$

Using the above-mentioned assumptions, the non-dimensional governing equations can be written as follows:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{1}$$

$$U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + \frac{1}{Re}\left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2}\right) \tag{2}$$

$$U\frac{\partial V}{\partial X} + V\frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + \frac{1}{Re}\left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2}\right) + Ri\,\theta \tag{3}$$

$$U\frac{\partial \theta}{\partial X} + V\frac{\partial \theta}{\partial Y} = \frac{1}{Re\,Pr}\left(\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2}\right) \tag{4}$$

The energy equation for solid region is:

$$\frac{\partial^2 \theta_s}{\partial X^2} + \frac{\partial^2 \theta_s}{\partial Y^2} = 0 \tag{5}$$

The non-dimensional parameters that found in the above formulation are the Reynolds number $\left(Re = V_0 L/\upsilon\right)$, Prandtl number $\left(Pr = \nu/\alpha\right)$, Richardson number $\left(Ri = g\beta\Delta T L\big/V_0^{\,2}\right)$ and solid fluid thermal conductivity ratio $\left(K = k_s/k_f\right)$, respectively.

The boundary conditions used in the present study are:

at the left vertical wall: $U = 0, V = 1,\ \theta = 0$

at the right vertical wall: $U = 0, V = 0,\ \theta = 0$

at the top and bottom walls: $U = 0,\ V = 0,\ \dfrac{\partial \theta}{\partial N} = 0$

at the inner surface of the cylinder: $U = 0, V = 0,\ \theta = 1$.

at the outer surface of the cylinder:

$$\begin{cases} U = 0, V = 0 \\ \left(\dfrac{\partial \theta}{\partial N}\right)_{fluid} = K\left(\dfrac{\partial \theta_s}{\partial N}\right)_{solid} \end{cases}$$

Where N is the non-dimensional distances either along X or Y direction acting normal to the surface. The average Nusselt number at the heated surface of the cylinder based on the dimensionless quantities may be expressed by $Nu = -\frac{1}{\pi}\int_0^{\pi}\frac{\partial \theta}{\partial n}d\varphi$ and the average temperature of the fluid in the cavity is defined by $\Theta = \int \theta\, d\overline{V}/\overline{V}$, where n represents the unit normal vector on the surface of the cylinder and $\overline{V}$ is the cavity volume. The stream function is calculated from its definition as

$$U = \frac{\partial \psi}{\partial Y},\ V = -\frac{\partial \psi}{\partial X}.$$

3.1 Solution Procedure

In this study, the Galerkin weighted residual method of finite-element formulation is used as a numerical procedure. The finite-element method begins by the partition of the continuum area of interest into a number of simply shaped regions called elements. These elements may be different shapes and sizes. Within each element, the dependent variables are approximated using interpolation functions. In the present study, erratic grid size system is considered especially near the walls to capture the rapid changes in the dependent variables. The coupled governing equations (2)-(5) are transformed into sets of algebraic equations using a finite-element method to reduce the continuum domain into discrete triangular domains. The system of algebraic equations is solved by iteration technique. The solution process is iterated until the subsequent convergence condition is satisfied: $\left|\Psi^{m+1}-\Psi^{m}\right| \le 10^{-4}$ where m is number of iteration and Ψ is the general dependent variable.

4. RESULTS AND DISCUSSION

A numerical analysis of lid driven mixed convection in a square enclosure wit a heated square hollow cylinder has been made using Galerkin weighted residual method of finite-element technique.

The study is made for different location of hollow cylinder in the lid-driven cavity for pure forced convection and mixed convection with *Ri* = 0.0, and 1.0, respectively, which influence the flow fields and temperature distribution inside the cavity. Air was used as working fluid inside the cavity with Pr = 0.71.

In this study, the characteristics of the flow and thermal field in the lid driven cavity are examined by exploring the effects of Richardson number, and position of the square heated hollow cylinder in the cavity. The dependence of flow and thermal fields for different locations of the heated cylinder can be observed in the plots of streamlines and isotherms in Figs. 2-3.

The effect of different location of the heated cylinder on streamlines is depicted in Fig. 2. From Fig. 2(a), it can easily be seen that a clockwise (CW) vortex is developed near the left vertical wall in the pure forced convection region (*Ri* = 0.0), due to the motion of the left wall. When the heated cylinder is moved from the position A to B, it is found that the flow strength of the core is same but the shape changes. One mat notice that the core of the vortex changes from circular to elliptic at C. However, a small CW eddy appears near the right side of the cylinder in the mixed convection region (Ri = 1.0) as shown in Fig. 2(b). It is clearly seen that in the mixed convection region (*Ri* = 1 0),) the major vortex inside the cavity is divided into two CW vortices with different flow strength while the cylinder is positioned at C. But while the cylinder moves closer to the right vertical wall at the position D, number of rotating cell increases.

The corresponding effect of thermal fields on the locations of the heat-generating block in the cavity can be obtained in Fig. 3. It is observed that the isothermal lines near the heat source are almost parallel to the nearest vertical wall due to the dominating influence of the conduction and mixed convection heat transfer. One may notice that higher values isotherms seem to be elliptic rounding the heated hollow cylinder. An interesting observation is that the isotherm turned back to the bottom surface for the location B (0.5, .25)

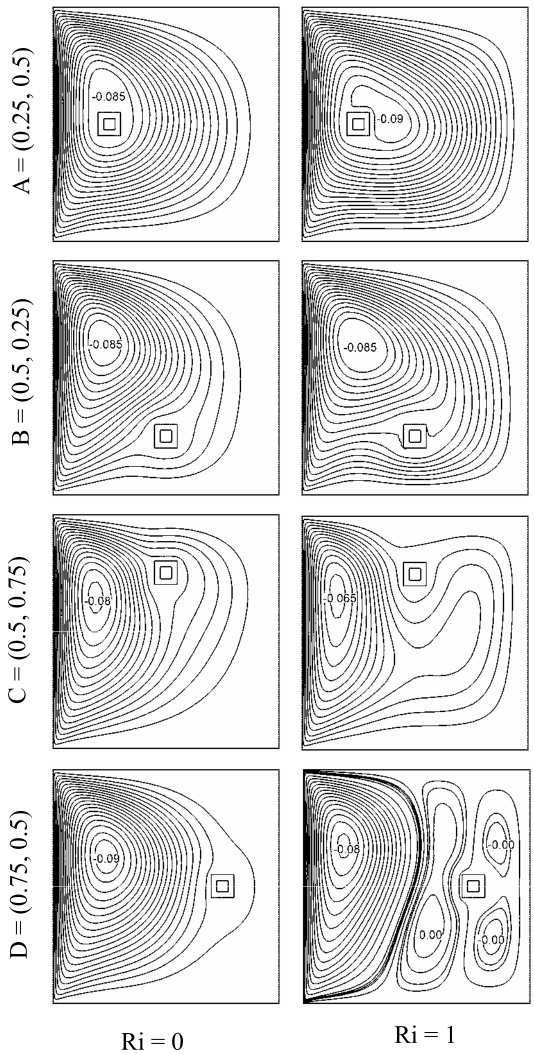

Fig 2. Streamlines for (a) Ri = 0 and (b) Ri = 1 at selected values of cylinder location.

The influence of cylinder location on average Nusselt number *Nu* at the heated surface and average temperature of the fluid in the cavity as a function of *Ri* is presented in Figs.4 (a)-(b). It can easily be seen that the values of average Nusselt number *Nu* decrease when the heated hollow cylinder is moved from the position A to B for both cases of Ri. Again Nu increase when it moves from the position B to C. Later, it decreases. However, highest average Nusselt number *Nu* is recorded for the lowest Ri = 0.0 when the cylinder is positioned at C. On the other hand, average fluid temperature in the cavity is higher for Ri=1 than Ri=0 at the position A. The maximum average fluid temperature in the cavity is found at the position C for both cases of Ri.

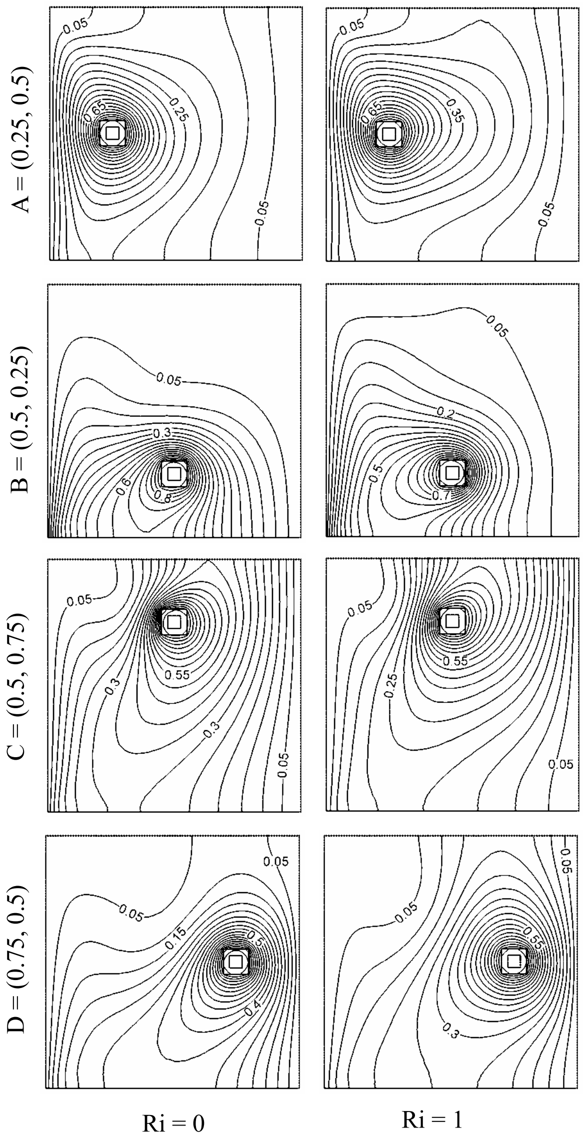

Fig. 3. Isotherms for (a) Ri = 0 and (b) Ri = 1 at selected values of cylinder location.

Figs. 5(a)-(b) illustrates the outcome of the location of the heated hollow cylinder on the drag force and temperature gradient respectively. The drag force increases when the location is changed from A to B as shown in Fig. 5(a). It is observed that maximum drag force attains at B for both cases. After that it decreases for all locations. On the other hand, the lowest temperature gradient is found when the cylinder is placed at B. But the maximum temperature gradient occurs while the cylinder is placed at C for both situations.

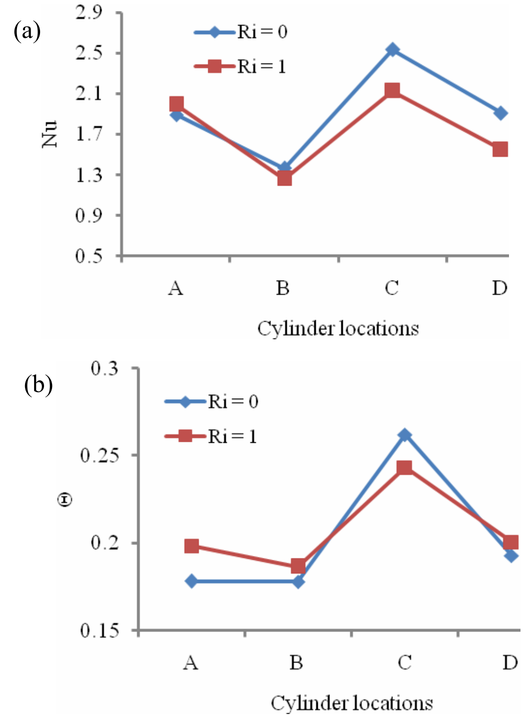

Fig 4. (a) Average Nusselt number, (b) average fluid temperature in the cavity versus cylinder location

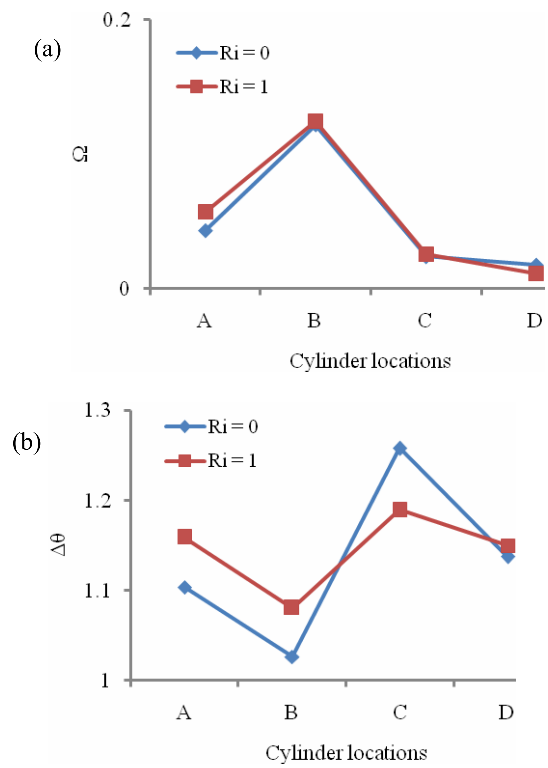

Fig 5. (a) Drag force, and (b) temperature gradient in the domain versus cylinder location.

5. CONCLUSIONS

Galerkin weighted finite-element technique is used to investigate the present problem. Two–dimensional, steady, laminar mixed convection in a lid-driven cavity containing a square hollow cylinder have been investigated numerically for different locations of the cylinder in the cavity, and Richardson number *Ri*. From an investigation of the heat transfer and fluid flow phenomena revealed by the numerical experiments, the following major result has been found as follows.

(a) The location of the square heated hollow cylinder can be used as a control parameter for heat transfer, fluid flow and temperature distribution.

(b) The position of the cylinder has a significant influence on the flow field in the pure mixed region in the cavity. Heat transfer is also strongly influenced by the placement of the hollow cylinder.

(c) Maximum heat transfer rate is observed while the cylinder is placed at C for both situations Ri = 0 and 1.

6. REFERENCES

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7. NOMENCLATURE

Symbol	Meaning	Unit
g	gravitational acceleration	ms^{-2}
Gr	Grashof number	
k_f	thermal conductivity of fluid	$Wm^{-1}K^{-1}$
k_s	thermal conductivity of the solid obstacle	$Wm^{-1}K^{-1}$
K	Solid fluid thermal conductivity ratio	
L	length of the cavity	M
Nu	average Nusselt number	
p	dimensional pressure	Nm^{-2}
P	dimensionless pressure	
Pr	Prandtl number	
Re	Reynolds number	
Ri	Richardson number	
T	dimensional temperature	K
ΔT	dimensional temperature difference	K
u, v	dimensional velocity components	ms^{-1}
U, V	dimensionless velocity components	
V_0	velocity of moving lid	ms^{-1}
$\bar{V}$	cavity volume	m^3
x, y	Cartesian coordinates	m
X, Y	dimensionless Cartesian coordinates	
	Greek symbols	
α	thermal diffusivity	m^2s^{-1}
β	thermal expansion coefficient	K^{-1}
γ	penalty constraint	
ν	kinematic viscosity	m^2s^{-1}
θ	non dimensional temperature	
ρ	density of the fluid	kgm^{-3}
μ	dynamic viscosity of the fluid	m^2s^{-1}
ψ	stream function	
φ	angular displacement from the front stagnation point, degrees	
	Subscripts	
h	heated wall	
c	cold	
s	solid surface	